

Related Rates Assignment

6/29/2020

1. $V = \frac{4}{3}\pi r^3$, given $\frac{dV}{dt} = 12 \text{ in}^3/\text{s}$, $r = 2 \text{ in}$

Differentiating w.r.t t implicitly

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \quad \dots (i)$$

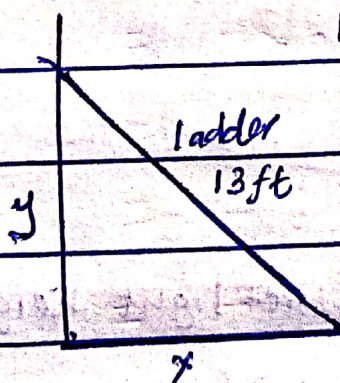
Substituting given values to equation (i)

$$12 = 4\pi (2)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{12}{4\pi \cdot 4}$$

$$\frac{dr}{dt} = \frac{3}{4\pi}$$

Q.

Let x be distance between wall base & ladderLet y be height of ladder off the ground

$$\frac{dx}{dt} = +4 \text{ ft/min, at moment } x = 5 \text{ ft}$$

Using pythagoras theorem: $x^2 + y^2 = 13^2 \quad \dots (a)$ Implicitly differentiate (a) above $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$$2x \frac{dx}{dt} = -2y \frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$x = 5, \quad \frac{dx}{dt} = 4$$

$$x^2 + y^2 = 13^2$$

$$5^2 + y^2 = 13^2$$

$$y^2 = 13^2 - 5^2$$

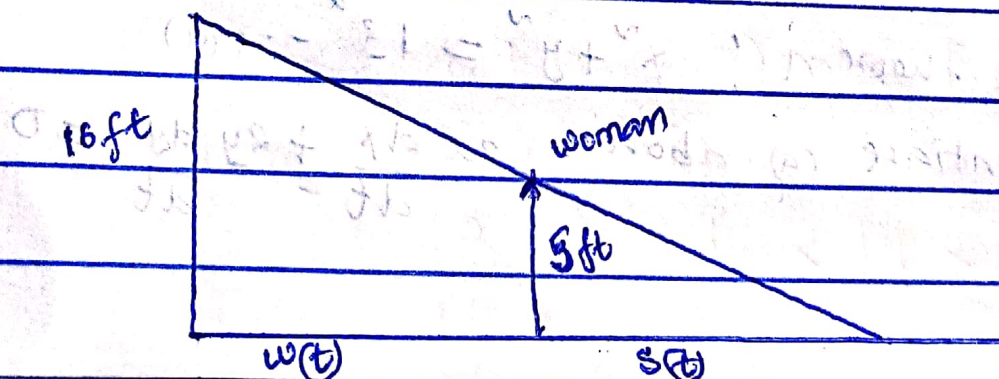
$$y^2 = 144$$

$$y = 12$$

$$\frac{dy}{dt} = -\frac{5}{12} \cdot 4$$

$$= -\frac{5}{3} \text{ ft/minute}$$

3. Given: Woman height = 5 ft, street light height = 16 ft
rate = 3 ft/sec, at moment 10 ft



$$\frac{s(t)}{5} = \frac{s(t) + w(t)}{16}$$

$$16 s(t) = 5 s(t) + 5 w(t)$$

$$11 s(t) = 5 w(t)$$

$$w(t) = \frac{11}{5} s(t) \dots (a)$$

$$\frac{dw(t)}{dt} = 3 \frac{\text{feet}}{\text{second}}$$

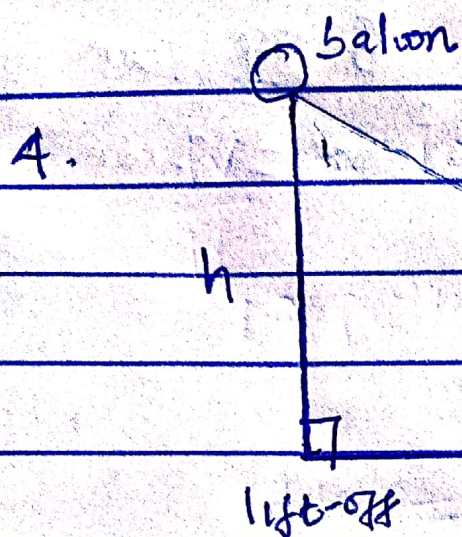
$$\frac{dw(t)}{dt} = \frac{d\left(\frac{11}{5} s(t)\right)}{dt}$$

$$\frac{dw(t)}{dt} = \frac{11}{5} \frac{ds(t)}{dt}$$

$$\frac{3 \times 5}{11} = \frac{ds(t)}{dt}$$

$$\frac{ds(t)}{dt} = \frac{15}{11} \text{ ft/sec}$$

$$= \frac{3}{2} \text{ ft/sec}$$



Let h be balloon height from lift-off.

θ be angle of elevation.

$\frac{dh}{dt} = 40 \text{ ft/min}$, Find $\frac{d\theta}{dt}$ at 30 minutes

$$\tan \theta = \frac{h}{500}$$

differentiating w.r.t time

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{500} \frac{dh}{dt} \quad \text{--- (i)}$$

At 30 minutes, $h = 40 \text{ ft/min} \times 30 \text{ min}$
 $= 1200 \text{ ft}$

$$\tan \theta = \frac{1200 \text{ ft}}{500 \text{ ft}} = 2.4$$

$$\theta = \arctan(2.4) = 67.38^\circ$$

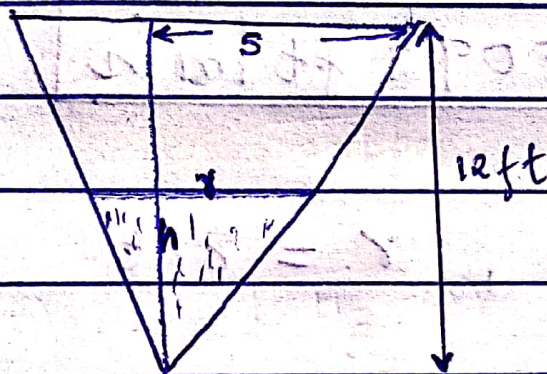
Rewriting equation (i)

$$\frac{d\theta}{dt} = \frac{1}{500} \times \frac{dh}{dt} \times \frac{1}{\sec^2 \theta}$$

$$\frac{d\theta}{dt} = \frac{1}{500} \times 40 \times \frac{1}{2.5913} \text{ min}^{-1} = 1.1$$

$$= \boxed{0.0309^\circ/\text{min}}$$

5.



Volume of con, $V = \frac{1}{3} \pi r^2 h$

Using similar triangles $\frac{5}{12} = \frac{r}{h} \Rightarrow 5h = 12r$

$$r = \frac{5}{12} h \quad \text{--- (a)}$$

Substituting (a) into volume equation

$$\begin{aligned} V &= \frac{1}{3} \pi \left(\frac{5}{12} h \right)^2 h \\ &= \frac{25\pi}{432} h^3 \end{aligned}$$

Differentiating w.r.t t

$$\frac{dV}{dt} = \frac{25\pi}{432} h^2 \frac{dh}{dt}$$

$$\frac{dv}{dt} = 10 \text{ ft}^3/\text{min}$$

$$10 \text{ ft}^3/\text{min} = \frac{25\pi (6 \text{ ft})^2}{144} \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{10 \times 144}{36 \times 25\pi}$$

$$= \boxed{0.5092 \text{ ft/min}}$$

$$6. \quad \frac{dr}{dt} = 2 \text{ ft/sec}, \quad r = 8 \text{ ft}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2r\pi \frac{dr}{dt}$$

$$\left. \frac{dA}{dt} \right|_{r=8} = 2 \times 8 \times \pi \times 2$$

$$= \boxed{32\pi \text{ ft}^2/\text{sec}}$$